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1 Basic Concept

1.1 SNR

SNR, the signal noise ratio is defined as the ratio of power of a signal to the power of background noise

$$SNR = \frac{P_{signal}}{P_{noise}}$$

SNR in decibels

$$P_{signal,dB} = 10log_{10}(P_{signal})P_{noise,dB} = 10log_{10}(P_{noise})$$

SNR in dB is

$$\begin{aligned} &10log_{10}(\frac{P_{signal}}{P_{noise}}) \\ =&10log_{10}(P_{signal}) - 10log_{10}(P_{noise}) \\ =&P_{signal,dB} - P_{noise,dB} \end{aligned}$$

We see by using the unit of dB, the ratio of the power turns into addition or subtraction.

2 Nonlinear Effect

Nonlinear effect means when a signal is passed into a device, the relationship between the output and input is not linear. This article aims to provide an explanation of the nonlinear process.

2.1 Harmonics

Suppose we have an input signal with a voltage

$$v_i(t) = V_{im}cos(\omega_i t)$$

The output signal can be approximated by Taylor's expansion

$$\begin{aligned} v_o(t) &= a_1 V_{im} cos(\omega_i t) + a_2 V_{im}^2 cos^2(\omega_i t) + a_3 V_{im}^3 cos^3(\omega_i t) \\ &= \frac{1}{2} a_2 V_{im}^2 + (a_1 V_{im} + \frac{3}{4} a_3 V_{im}^3) cos(\omega_i t) + \frac{a_2}{2} V_{im}^2 cos(2\omega_i t) \end{aligned}$$

All the terms with $cos(n\omega_i t)$ are nonlinear terms. Therefore, an input signal with frequency ω has output with frequency $n\omega$, and this is called nonlinear effect. All the signal component with frequency $n\omega$ are called harmonics.

2.2 Gain compression

In the above derivation, the amplitude of $cosw_it$ is $a_1V_{im} + \frac{3}{4}a_3V_{im}^3$ is the gain to the input signal g_m. In most cases $a_3 < 0$, when v_{im} is sufficiently small, the first term dominates so the gain is equal to $log(a_1)$. But when v_m is large enough, the second term is not negligible so the gain decreases from $log(a_1)$. We define **1db compression point** as the input power when the gain is decreased by 1dB from $log(a_1)$

2.3 Intermodulation

Suppose we have two input signals

$$V_{in}(t) = V_{1m}cos(\omega_1 t) + V_{2m}cos(\omega_2 t)$$

The third order output power term $V_{out}^{(3)}$has two components

$$\begin{aligned} V_{out}^{(3)} =& \frac{3}{4}a_3V_{1m}^2V_{2m}cos(2\omega_1 + \omega_2)t \\ &+ \frac{3}{4}a_3V_{1m}^2V_{2m}cos(2\omega_1 - \omega_2)t \\ &+ \frac{3}{4}a_3V_{1m}V_{2m}^2cos(2\omega_2 + \omega_1)t \\ &+ \frac{3}{4}a_3V_{1m}V_{2m}^2cos(2\omega_2 - \omega_1)t \end{aligned}$$

Since these components come from sources with two frequencies ω_1 and ω_2, we call it **intermodulation**. In general, the output contain components at frequencies such as $k_1\omega_1 + k_2\omega_2$. The order O of the intermodulation is defined as the sum of the coefficients in the frequencies

$$O = \sum_i |k_i|$$

So we call $V_{out}^{(3)}$ as **third order intermodulation power** because $|k_1|+|k_2| = 3$. if $V_{1m} = V_{2m} = V_m$

$$\begin{aligned} V_{out}^{(3)} &= \frac{3}{4}a_3V_m^3[cos(2\omega_1 - \omega_2)t + cos(2\omega_2 - \omega_1)t] \\ &+ \frac{3}{4}a_3V_m^3[cos(2\omega_1 + \omega_2)t + cos(2\omega_2 + \omega_1)t] \end{aligned}$$

We are in particular interested in the components at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$, because as ω_1 approaches ω_2, they fall within the vicinity of the original frequency components, and may therefore interfere with the desired behavior. The intermodulation distortion(**IMD**) is the ratio of the output power at the combination frequency to the output power at the fundamental. For example, if $V_{1m} = V_{2m} = V_m$

$$\begin{aligned} V_{out}^{(3)} &= \frac{3}{4}a_3V_m^3[cos(2\omega_1 - \omega_2)t + cos(2\omega_2 - \omega_1)t] \\ &+ \frac{3}{4}a_3V_m^3[cos(2\omega_1 + \omega_2)t + cos(2\omega_2 + \omega_1)t] \end{aligned}$$

For combination frequency $|2\omega_1 - \omega_2|$ and the fundamental ω_1, the **IMD** is

$$IMD = (\frac{3a_3V_m^2}{4a_1})^2$$

2.4 3rd order intercept point

The output power at base frequency

$$P_{o1} = \frac{1}{2}(a_1V_m)^2 \equiv= G_1P_i$$

The output power at intermodulation frequency

$$P_{o3} = \frac{1}{2}(\frac{3}{4}a_3V_m^3)^2 \equiv= G_3P_i^3$$

So

$$P_{o1} = 10logG_1 + 10logP_i$$
$$P_{o3} = 30logG_3 + 30logP_i$$

So both P_{o1} and P_{o3} has a linear relationship with P_i. The above equations represent two lines, but the slope of P_{o3} is 3 times of that of P_{o1}. The third-order intercept point(IP3) is defined as the intercept point of the above two lines(P_{o1} and P_{o3}). We can use the slope relation to find the intercept point.

$$\frac{IP_3 - P_{o3}}{IP_3 - P_{o1}} = 3$$

Therefore

$$IP_3 = \frac{1}{2}(3P_{o1} - P_{o3})$$