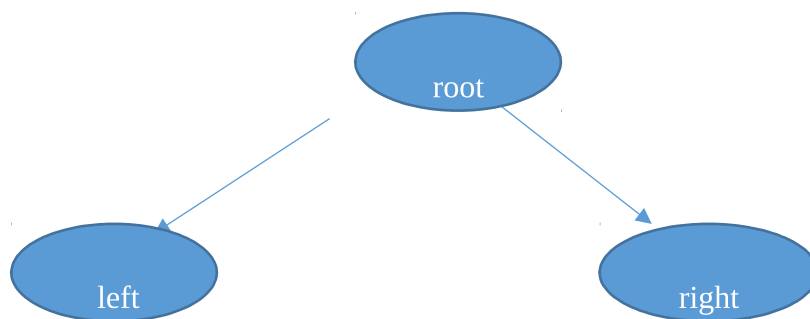
Heap

Heap belongs to a generic tree data structure with a special property that:
1) It is a complete binary tree, which means all levels are completely filled except possibly the last level and the last level has all keys as left as possible
2) for every node, it has to be larger than its children for max heap and has to be smaller than its children for min heap.

Representation: We can use the usual representation of tree definition which is for each node we define its left and right child reference. And there is also an alternative way of doing it. Since heap is complete tree, so we can store its element to an array by its level traversal. For an element with index i, its left child index is 2i+1 and 2i+2.

1. **Keep the heap property(assuming it is all max heap)**

In order to maintain heap structure, we define a heapfy function.
Function: heapfy()
a. Algorithm:



1) If left > root, swap(left, root), or if right > root, swap(right, root)
2) Recursively call heapfy() on the swap the child node till end so the heap property is reserved.

b. Code

```
// n size of array, i is start index.
Void heapfy(int[] arr, int n, int i)
{
    int largest = i;
      int l = 2i+1;   // binary tree index relations
      int r = 2i+2;
      if(l<n && arr[l] > arr[largest])
        largest = i;
      if(r<n && arr[r]> arr[largest])
          largest = r;
     //swap
      If(largest != i)
      {
        swap(arr[i], arr[largest]);
          heapfy(arr, n, largest);
       }
}
```

c. Time complexity

h is the height of the tree

O(h) = O(log_2 n)

2. **Build Heap**

a. Algorithm

do heapfy() on all the nodes that have children, these nodes have index(n/2-1, 0) (based on binary tree property)

b. Code

```
void buildHeap(int[] arr, int n)
{
   for(int i=n/2  -1; i>=0; i--)
   {
      heapfy(arr, n, i);
   }
}
```

c. Time Complexity

Time = \sum _{h=0} ^ {log_2n} [\frac{n}{2^{h+1}}]
O(h)

O(h) : time to call one heapfy()

$[\frac{n}{2^{h+1}}]$: number of nodes on the same tree level

$\sum _{h=0} ^ {log_2^n}$: sum over all the levels.

So time = $O(n \sum_{h=0}^{(lgn)} h/(2^h))$
= O(2n)
=O(n)

3. Heapsort

a. algorithm

1) Build the heap
2) Start from the last element(index n-1),
3) swap it with the first element. Then the last element becomes sorted
4) Do heapfy on the unsorted elements
5) Move to the next unsorted element, in this case it is the last but two(index n-2), repeat 3) and 4) until all elements become sorted.

b. Code

```
Void heapsort(int arr[], int n)
// build heap
void buildHeap(arr, n);
for(int i = n-1; i>=0; i--)
{
  swap(arr[0], arr[i]);
  heapfy(arr, i, 0);
}
```

c. Time complexity

1)Build heap O(n)
2)Sort nlog_2 n

d. Comparison

1) Compare to quicksort

	QS	HS	comment
Average Speed	Faster		12nlogn vs 16 nlog n
Worst Speed		Faster	n^2 vs nlogn

2) Compare to mergesort

	MS	HS	comment
Space	O(n)	O(1)	MS needs additional space for merge
Stable **s**ort	Yes		MS keeps order for same element
Better Cache performance	Yes		MS accesses the caches that are near to each other.

4. Binary heap time complexity analysis

a. Time complexity table

	Average	Worst
Search	O(n)	O(n)
Insert	O(1)	O(h) = O(long(n))
Delete	O(log(n))	O(long(n))
Peek	O(1)	O(1)

b. Insert

1) Add the element to the bottom level of the heap

2) Compare the added element with its parent
If they are in correct order, stop
3) If not swap the element with its parent and return to the 2) by keeping comparing the parent

c. Insert average time complexity

Assuming a uniform distribution of numbers, which means for any element in the heap, it has a one-half chance of being greater than its parent. And it has one-fourth chance of being greater than its grandparent. So the expected number of swap during the insertion is Probability of swapping with 1st parent * number of swap + Probability of swapping with 2st parent * number of swap + …+ Probability of swapping with m^{st} parent

= ½ * 1 + ¼ * 2 + 1/8* 3 + ½^m * m
= \sum \frac{m}{2^m}
= 2 when m goes to \inf

Therefore the averaged time complexity is O(1)

5. Priority Queue

a. Definition

PriorityQueue is a data structure that stores the data based upon their priority. For example, let us image we have a l ist of integers and we would like the data structure(called priority queue)to retrieve the max integer in constant time. Then we can implement this use max heap.

b. Property and usage

1) Data stored in the priority queue does not have to sorted, it only needs to maintain heap data structure. ie. parent is larger than children.
2) In order to maintain first K minimum data of a given data source, we use max heap. This is a little counter intuitive and the reason is we use max heap so that we can pop out the largest element when the queue reaches its capacity. Each time we add the element into the queue and maintain the heap structure, when we reaches the capacity of the queue, we pop out the largest element which is the root of the heap, and then maintain the heap structure again.

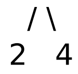
c. Examples

data: 3 2 4 5 4 and build a max heap

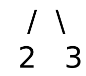
(1) 3 root node

(2) 3 add 2, max heap property is auto maintained.

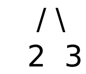
(3) 3 add 4, max heap property is violated.

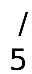


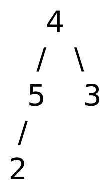
4 fix max heap property



(4) 4 add 5, max heap property is violated







(5) 5 add 4, max heap property is maintained